

7 Ratio and Proportion

7.1 Equivalent Ratios

Orange squash is to be mixed with water in a ratio of 1 : 6; this means that for every unit of orange squash, 6 units of water will be used. The table gives some examples:

<i>Amount of Orange Squash (cm³)</i>	<i>Amount of Water (cm³)</i>
1	6
20	120
5	30

The ratios 1 : 6 and 20 : 120 and 5 : 30 are all equivalent ratios, but 1 : 6 is the *simplest* form.

Ratios can be simplified by dividing both sides by the same number: note the similarity to fractions. An alternative method for some purposes, is to reduce to the form 1 : n or n : 1 by dividing *both* numbers by either the left-hand-side (LHS) or the right-hand-side (RHS). For example:

$$\text{the ratio } 4 : 10 \text{ may be simplified to } \frac{4}{4} : \frac{10}{4} \Rightarrow 1 : 2.5$$

$$\text{the ratio } 8 : 5 \text{ may be simplified to } \frac{8}{5} : \frac{5}{5} \Rightarrow 1.6 : 1$$



Example 1

Write each of these ratios in its simplest form:

(a) 7 : 14

(b) 15 : 25

(c) 10 : 4



Solution

(a) Divide both sides by 7, giving

$$\begin{aligned} 7 : 14 &= \frac{7}{7} : \frac{14}{7} \\ &= 1 : 2 \end{aligned}$$

(b) Divide both sides by 5, giving

$$\begin{aligned} 15 : 25 &= \frac{15}{5} : \frac{25}{5} \\ &= 3 : 5 \end{aligned}$$

(c) Divide both sides by 2, giving

$$\begin{aligned} 10 : 4 &= \frac{10}{2} : \frac{4}{2} \\ &= 5 : 2 \end{aligned}$$



Example 2

Write these ratios in the form $1 : n$.

(a) $3 : 12$

(b) $5 : 6$

(c) $10 : 42$



Solution

(a) Divide both sides by 3, giving

$$3 : 12 = 1 : 4$$

(b) Divide both sides by 5, giving

$$\begin{aligned} 5 : 6 &= 1 : \frac{6}{5} \\ &= 1 : 1.2 \end{aligned}$$

(c) Divide both sides by 10, giving

$$\begin{aligned} 10 : 42 &= 1 : \frac{42}{10} \\ &= 1 : 4.2 \end{aligned}$$



Example 3

The scale on a map is $1 : 20\,000$. What actual distance does a length of 8 cm on the map represent?



Solution

$$\begin{aligned} \text{Actual distance} &= 8 \times 20\,000 \\ &= 160\,000 \text{ cm} \\ &= 1600 \text{ m} \\ &= 1.6 \text{ km} \end{aligned}$$



Exercises

1. Write each of these ratios in its simplest form:

(a) $2 : 6$

(b) $4 : 20$

(c) $3 : 15$

(d) $6 : 2$

(e) $24 : 4$

(f) $30 : 25$

(g) $14 : 21$

(h) $15 : 60$

(i) $20 : 100$

(j) $80 : 100$

(k) $18 : 24$

(l) $22 : 77$

2. Write in the form $1 : n$, each of the following ratios:

(a) $2 : 5$

(b) $5 : 3$

(c) $10 : 35$

(d) $2 : 17$

(e) $4 : 10$

(f) $8 : 20$

(g) $6 : 9$

(h) $15 : 12$

(i) $5 : 12$

3. Write in the form $n : 1$, each of the following ratios:
- (a) $24 : 3$ (b) $4 : 5$ (c) $7 : 10$
(d) $15 : 2$ (e) $18 : 5$ (f) $6 : 5$
4. Jennifer mixes 600 ml of orange juice with 900 ml of apple juice to make a fruit drink. Write the ratio of orange juice to apple juice in its simplest form.
5. A builder mixes 10 shovels of cement with 25 shovels of sand. Write the ratio of cement to sand:
- (a) in its simplest form,
(b) in the form $1 : n$,
(c) in the form $n : 1$,
6. In a cake recipe, 300 grams of butter are mixed with 800 grams of flour. Write the ratio of butter to flour:
- (a) in its simplest form,
(b) in the form $1 : n$,
(c) in the form $n : 1$.
7. In a school there are 850 pupils and 40 teachers. Write the ratio of teachers to pupils:
- (a) in its simplest form, (b) in the form $1 : n$.
8. A map is drawn with a scale of $1 : 50\,000$. Calculate the actual distances, in km, that the following lengths on the map represent:
- (a) 2 cm (b) 9 cm (c) 30 cm.
9. A map has a scale of $1 : 200\,000$. The distance between two towns is 60 km. How far apart are the towns on the map?
10. On a map, a distance of 5 cm represents an actual distance of 15 km. Write the scale of the map in the form $1 : n$.

7.2 Direct Proportion

Direct proportion can be used to carry out calculations like the one below:

If 10 calculators cost £120,
then 1 calculator costs £12,
and 8 calculators cost £96.